ESTIMATION OF TIME VARIABLE PARAMETERS OF MACROECONOMIC MODEL WITH RATIONAL EXPECTATION

Tonner J.[1], Vašíček 0.[1], Štecha J.[2], Havlena V.[2]

[1]Department of Applied Mathematics and Computer Science
Faculty of Economics and Administration
Masaryk University
[2] Department of Control Engineering
Faculty of Electrical Engineering
Czech Technical University

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Outline

- Main features
- Introduction
- Hansen model
- Bootstrap filter
- Description of results
- Conclusions

Introduction

Problem statement and model

- Time - variant parameters estimation by Bootstrap filter
- Dynamic Stochastic General Equilibrium Model with Rational Expectation

Why to estimate time - variant parameters?

- Pure econometric approach or capricious nature.
- Parameters drifting as a character of economic environment.
- Parameters drifting as a telltale of model misspecification.

Our approach coincides with the second way.

Hansen model

$$\max \mathsf{E} \sum_{t=0}^{\infty} \beta^t [\ln(C_t) - \gamma H_t]$$

under four conditions.

- $Y_t = A_t K_t^{\theta} (\eta^t H_t)^{1-\theta}$
- $A_t = A^{1-\rho} A_{t-1}^{\rho} \mathsf{e}^{\epsilon_t}$ $\epsilon \sim \mathsf{N}(0, \sigma^2), \mathsf{cov}(\epsilon_i, \epsilon_j) = 0, i \neq j.$
- $Y_t = C_t + I_t$
- $K_{t+1} = (1 - \delta) K_t + I_t$

After optimization where the representative consumer chooses $\{Y_t, C_t, I_t, H_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize utility subject to the four conditions, the system attains the form

$$\begin{aligned} y_t &= a_t + \theta k_t + (1-\theta)h_t \\ a_t &= \rho a_{t-1} + \epsilon_t \\ \kappa y_t &= (\kappa - \theta\lambda)c_t + \theta\lambda i_t \\ \eta k_{t+1} &= (1-\delta)k_t + \lambda i_t \\ c_t + h_t &= y_t \\ -\frac{\eta}{\beta}c_t &= -\frac{\eta}{\beta}E_t c_{t+1} + \kappa E_t y_{t+1} - \kappa k_{t+1} \end{aligned}$$

where $\kappa = \frac{\eta}{\beta} - 1 + \delta$ and $\lambda = \eta - 1 + \delta$.

(For more details see Ireland, 2004, or Maley, 2004).

Hansen model

Or in matrix form

$$G \begin{bmatrix} x_{1,t+1} \\ E_t x_{2,t+1} \end{bmatrix} = D \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} + \begin{bmatrix} e_{t+1} \\ \mathbf{0} \end{bmatrix}, \text{ where}$$

$$x_{1,t} = \begin{bmatrix} k_t \\ a_t \end{bmatrix}, \quad x_{2,t} = \begin{bmatrix} y_t \\ i_t \\ h_t \\ c_t \end{bmatrix}, \quad e_t = \begin{bmatrix} 0 \\ \epsilon_{t+1} \end{bmatrix},$$

$$G = \begin{bmatrix} \eta & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -\kappa & 0 & \kappa & 0 & 0 & -\frac{\eta}{\beta} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1-\delta & 0 & 0 & \lambda & 0 & 0 \\ 0 & \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{\eta}{\beta} \\ -\theta & -1 & 1 & 0 & \theta-1 & 0 \\ 0 & 0 & \kappa & -\theta\lambda & 0 & \theta\lambda-\kappa \\ 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix}.$$

Hansen model

Using Generalized Shur Decomposition of matrices D and G we get $D = QSZ^T$ and $G = QTZ^T$. It is not difficult to show that resulting state model attains the form

$$x_{1,t+1} = \begin{bmatrix} k_{t+1} \\ a_{t+1} \end{bmatrix} = \Pi \begin{bmatrix} k_t \\ a_t \end{bmatrix} + \begin{bmatrix} 0 \\ \epsilon_{t+1} \end{bmatrix},$$

$$x_{2,t} = \begin{bmatrix} y_t \\ i_t \\ h_t \\ c_t \end{bmatrix} = U \begin{bmatrix} k_t \\ a_t \end{bmatrix},$$

where matrices Π and U are constructed from matrices Z, S, T:

$$\Pi = Z_{kl} S_{ll}^{-1} T_{ll} Z_{kl}^{-1}$$
$$U = Z_{jl} Z_{kl}^{-1}.$$

Indexes j, k, l denote appropriate submatrices corresponding with a number of stable roots (Söderlind, Paul 2003).

Ireland appends also VAR model, so

$$x_{1,t+1} = \begin{bmatrix} k_{t+1} \\ a_{t+1} \\ \vartheta_{y_{t+1}} \\ \vartheta_{c_{t+1}} \\ \vartheta_{h_{t+1}} \end{bmatrix} = \begin{bmatrix} \Pi & \mathbf{0} \\ \mathbf{0} & D \end{bmatrix} \begin{bmatrix} k_t \\ a_t \\ \vartheta_{y_t} \\ \vartheta_{c_t} \\ \vartheta_{h_t} \end{bmatrix} + \begin{bmatrix} 0 \\ \epsilon_{t+1} \\ \xi_{y_{t+1}} \\ \xi_{c_{t+1}} \\ \xi_{h_{t+1}} \end{bmatrix},$$

$$x_{2,t} = \begin{bmatrix} y_t \\ i_t \\ h_t \\ c_t \end{bmatrix} = \begin{bmatrix} U & I \end{bmatrix} \begin{bmatrix} k_t \\ a_t \\ \vartheta_{y_t} \\ \vartheta_{c_t} \\ \vartheta_{h_t} \end{bmatrix},$$

where matrix $D = \begin{bmatrix} d_{yy} & d_{yc} & d_{yh} \\ d_{cy} & d_{cc} & d_{ch} \\ d_{hy} & d_{hc} & d_{hh} \end{bmatrix}$ is matrix of correlation coefficients and I is identity matrix. (page 11) (page 13)

Bootstrap filter

Let us have non - linear discrete time system

$$\begin{aligned} x(t+1) &= f(x(t), u(t)) + v(t), \\ y(t) &= g(x(t), u(t)) + e(t), \end{aligned}$$

where $v(t) \sim p_v(v(t))$ and $e(t) \sim p_e(e(t))$ are white noises. Let us have CPDF

$$\begin{aligned} p(x(t+1)|x(t), u(t)) &= p_v(x(t+1) - f(x(t), u(t))) \\ p(y(t)|x(t), u(t)) &= p_e(y(t) - g(x(t), u(t))). \end{aligned}$$

Suppose we have the set of observed data

$$D^{t-1} = \{u(1), y(1), \dots, u(t-1), y(t-1)\}.$$

Bootstrap filter

Suppose the prior CPDF $p(x(t)|D^{t-1})$ is represented by an N-dimensional sample set

$$x^{(i)}(t|t-1) \sim p(x(t)|D^{t-1}), i = 1, \dots, N.$$

The samples can be considered as an empirical CPDF

$$p_N(x(t)|D^{t-1}) = \sum_{i=1}^{N} w^{(i)}(t|t-1)\delta(x(t) - x^{(i)}(t|t-1))$$

with uniform weights

$$w^{(i)}(t|t-1) = \frac{1}{N}, i = 1, \dots, N.$$

Then the knowledge of the state described by $p(x(t)|D^{t-1})$can be updated by the following steps:

Bootstrap filter

Data update step: Using $p(x(t)|D^{t-1})$ and the output measurement y(t) (7), $p(x(t)|D^t)$ is determined as

$$p(x(t)|D^t) = \frac{p(y(t)|x(t), u(t))}{p(y(t)|D^{t-1}, u(t))} p(x(t)|D^{t-1}),$$

or empirically as

$$p_N(x(t)|D^t) = \sum_{i=1}^{N} w^{(i)}(t|t)\delta(x(t) - x^{(i)}(t|t)),$$

where $x^{(i)}(t|t) = x^{(i)}(t|t-1)$.
The posterior weights are given by normalized likelihood

$$w^{(i)}(t|t) = \frac{p(y(t)|x(i)(t|t-1), u(t))}{\sum_{j=1}^{N} p(y(t)|x(j)(t|t-1), u(t))},$$

for $i = 1, \ldots, N$.

Bootstrap filter

Time update step is theoretically given by state transition equation

$$p(x(t+1)|x(t), D^t) = p(x(t+1)|x(t), u(t)),$$

or empirically by importance resampling of $p_N(x(t)|D^t)$ with additive noice $v(t)$, i.e.

$$x^{(i)}(t+1|t) = f(x^{(j)}(t|t), u(t)) + v^{(i)}(t), (7)$$

where $x^{(j)}(t|t)$ is a sample drawn from $p_N(x(t)|D^t)$ and $v^{(i)}(t)$ is sample drawn from $p(v(t))$.
The predicted, filtered and smoothed estimates of the state can be obtained as a weighted sample mean

$$\hat{x}_{MS}(t|t') = \sum_{i=1}^{N} w^{(i)}(t|t')x^{(i)}(t).$$

More detailed description can be found in Štecha and Havlena, 1995 or Havlena and Štecha, 1998 or Trnka, 2004.

The results of $p(x(1)|D^0)$ carried out by widely used software DYNARE (see Juillard) can be seen in the following table.

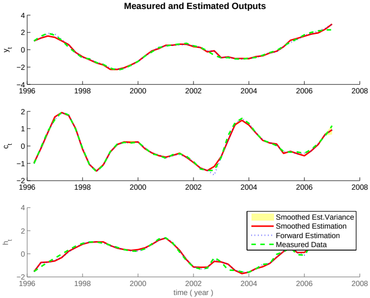
	prior mean	postmean	conf. interval-l	conf. interval-r	priorstd
β	0.990	0.992	0.988	0.998	0.005
γ	0.004	0.004	0.002	0.006	0.002
θ	0.200	0.267	0.164	0.388	0.070
η	1.004	1.002	0.976	1.027	0.015
δ	0.025	0.025	0.005	0.052	0.012
A	6.000	6.100	4.142	7.452	1.000
ρ	0.990	0.994	0.989	0.999	0.005
σ	0.390	0.276	0.205	0.355	0.070
d_{yy}	0.980	0.981	0.971	0.987	0.005
d_{yc}	0.000	-0.067	-0.143	0.010	0.100
d_{yh}	0.000	0.126	0.033	0.224	0.100
d_{cy}	0.000	0.093	0.011	0.159	0.100
d_{cc}	0.980	0.979	0.961	0.993	0.010
d_{ch}	0.000	0.104	-0.002	0.222	0.100
d_{hy}	0.000	-0.407	-0.492	-0.312	0.100
d_{hc}	0.000	-0.268	-0.348	-0.171	0.100
d_{hh}	0.300	0.304	0.286	0.321	0.010
v_y	0.280	0.353	0.284	0.443	0.080
v_c	0.460	0.450	0.370	0.520	0.110
v_h	0.002	0.002	0.001	0.002	0.001

Description of results

We carried out two experiments:

- time - variable parameters estimation with VAR model (page 7)
- time - variable parameters estimation without VAR model (page 6)

Time - variable parameters estimation with VAR model



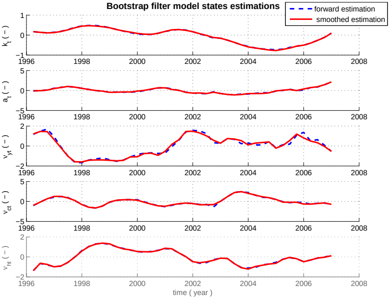
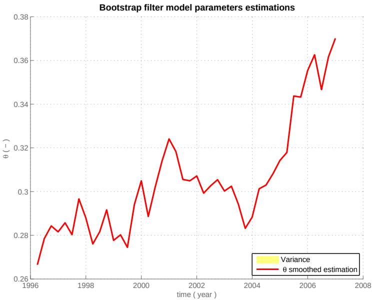


Fig. I. Outputs y - with VAR Fig. II. State x - with VAR

Description of results

Parameter θ was set time - variant.



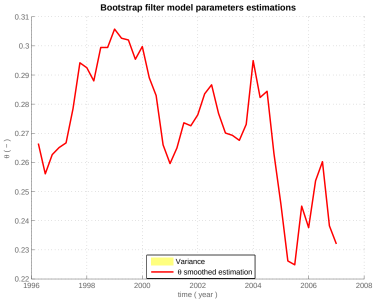
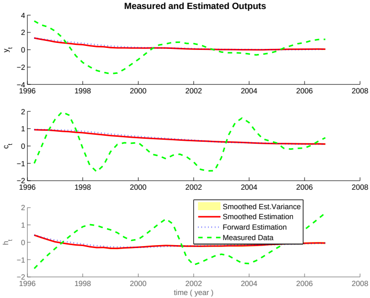


Fig. III. Parameter θ - with VAR Fig. IV. Parameter θ - with VAR

Description of results

Time - variant parameters estimation without VAR model



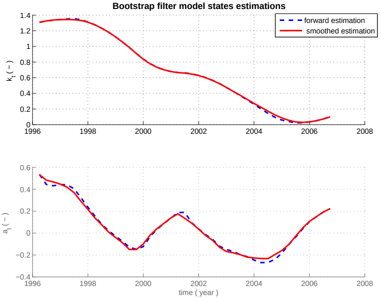


Fig. V. Outputs y - without VAR Fig. VI. State x - without VAR

Description of results

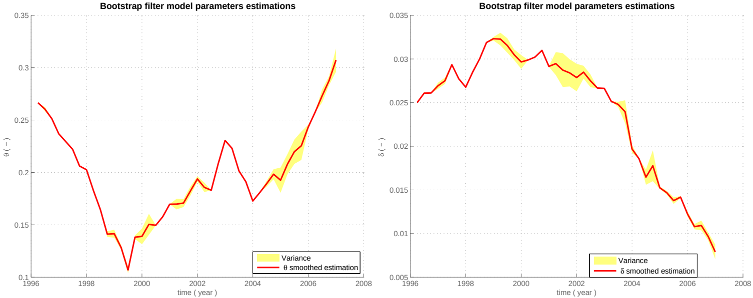
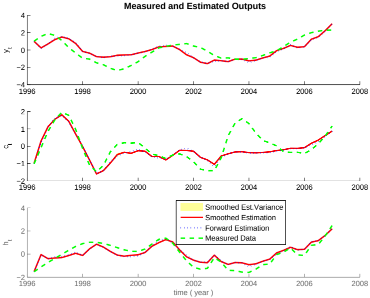


Fig. VII. Parameter θ - suppressed VAR
Fig. VIII. Parameter δ - suppressed VAR

Description of results



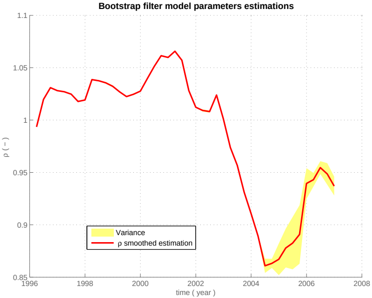


Fig. IX. Parameter ρ - suppressed VAR

Fig. X. Outputs y - suppressed VAR

Conclusions

- The method of time-variant parameter estimation by Bootstrap filter can be applied only to models which are well - determined.
- The Hansen model with VAR and with time - variant parameters is a typical example of overcomplete model.
- On the other hand, the poor base causes that we are not able to sufficiently fit the data (experiment without VAR). In this case we tried to complete the base with the time - variant parameters and with the suppressed VAR model.
- Our work also brings new theoretical contribution which embodies in solution of rational expectations problem for time-variant parameter estimation by Bootstrap filter.

Thank you for your attention

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jtonner@tiscali.cz

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