Geostatistische Modelle für Fließgewässer

Gregor Laaha (gregor.laaha@boku.ac.at)
Institute of Applied Statistics and Computing,
BOKU Vienna, Austria


Introduction: Spatial Interpolation
- Estimation at a certain location
  e.g. Air pollutant concentrations were measured at different locations.
  What is the concentration at location $X_o$?

Introduction: Grid-based Estimation
- Estimation of a value for each cell of a grid
- Presentation in form of a map (Mapping)
  Example: Air pollutant concentrations were measured at different locations.
  Air pollution map
**Simple Interpolation Methods**

**Example: Triangulation**

Plain through the next three data points (1,2,3)

**Calculation:** Delaunay method

- linear combination
- weightings of opposing area

\[
Z^* = \sum_\lambda \lambda^* Z = \lambda^1 Z_1 + \lambda^2 Z_2 + \lambda^3 Z_3
\]

**Weighting:**

\[
\lambda^1 = \frac{A_{132}}{A_{123}}, \quad \lambda^2 = \frac{A_{231}}{A_{123}}, \quad \lambda^3 = \frac{A_{312}}{A_{123}}
\]

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**From Simple Interpolation to Geostatistics**

**In a nutshell:**

- Estimation by linear combination of data points
- Unbiased estimator
- Empirical chosen weights (not optimal!)

**Optimal weights?**

- Estimation variance as a quality criteria
  \[
  \sigma^2 = \text{var}[Z^* - Z] = E[(Z^* - Z)^2]
  \]
  - Optimal estimation – minimum estimation variance
  \[
  \sigma^2 = \text{var}[Z^* - Z] = \text{min}!
  \]
  - Geostatistical interpolation (Kriging)

  Optimal weights on the basis of **spatial correlation**

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**Spatial correlation**

**Variogram (Semivariogram)**

... dissimilarity vs. distance (h)

... for pairs of data points

\[
\gamma(h) = \frac{(Z(x) - Z(y))^2}{2}
\]

- In case of stationary/intrinsic random field the variogram is only a function of distance h.

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**Geostatistics...**

**How for stream networks ??**
Introduction: The river network problem...

- Estimation of streamflow and related variables
  - fundamental problem in WRM
- Gauged sites
  - summary statistics of observed time series
- Ungauged sites?
  - regional transfer of observed information

Focus on geostatistical regionalisation methods
- spatial average, weights according to spatial covariance
- rarely used in practice

Challenge: Tree structure of river network
- catchments related to points of the river network are organised into subcatchments (i.e. they are nested)
- they need to be treated differently from flow-unconnected neighbours which do not share a catchment

Kriging on river networks – two concepts discussed:
- 1D models, 2D models
- Compared to OK-Euclid
1D Models

- Treat river network as 1D problem
- Support = river location
- Ordinary point-kriging predictor
- requires meaningful distance metric & valid Cov-Function

\[ \hat{z}(x_i) = \sum_{j=1}^{n} \lambda_j z(x_j) \]

Euklidean Stream distance


1D Models ... valid covariance function

- Gottschalk (1993a) first calculated covariance along stream network based on river distance
  - ... exponential Cov-Function well suited
  - ... added water balance constraints to kriging system to ensure predicted lateral inflow = difference b/w gauges
- Ver Hoef et al. (2006), Cressie et al. (2006)
  - Spatial Cov-Function C(h)
    - ... derived by moving average (kernel convolution)
    - ... different kernel shapes -> relate to different Cov-Functions
    - => classical Cov-Functions are valid for river networks
- Restriction: only unilateral kernels
  - ... downstream (Tail-down model) or
  - ... upstream (Tail-up model)

1D Models ... Pointkriging using stream distance

Ver Hoef et al. (2006) Tail-up model performs better
... but needs auxiliary variables for weighting confluents
  - catchment area (Ver Hoef, 2006);
  - stream order (Cressie, 2006)
... as surrogate of discharge (sic!)

-> ct. Gottschalk (1993a,b): discharge constraints
2D Models

- Runoff generation = continuous spatial process
  ... existing in any point of the landscape
- Discharge at river site = integral of point runoff over catchment
  \[ z(A) = \int_A z(x) \, dx \]
- Support = catchment area
- Regional transfer ("prediction") = Change of support
  ... Block-kriging
  ... irregular support (!)
  ... river network topology (!)
- Implementation not trivial, but consistent hydrological concepts of runoff generation

2D-Model Top-Kriging (Skøien et al. 2006)

- Regularised variogram
  ... spatial correlation b/w pairs of catchments
  - with different support (area)
- Variograms for pairs of catchments ... a function of distance (h) and support \((A_1, A_2)\)
  \[
  \gamma_{12}(h) = \gamma(h, A_1, A_2) - \frac{1}{2} \left[ \gamma(h, A_1, A_1) + \gamma(h, A_2, A_2) \right]
  \]
  ... is smaller for overlapping catchments
  => More weight to nested catchments

R-package rtop – see Poster P-033, J.O. Skøien et al.

Comparison of geostatistical models

- Kriging = spatial weighted average
  \[ \Rightarrow \text{Methods are as good as kriging weights} \]
  ... and how they are distributed in space
  \[ \Rightarrow \text{How are weights distributed b/w connected and unconnected neighbours?} \]
  \[ \Rightarrow \text{Focus on limiting situations} \]
  (i) equally distant neighbours
  (ii) more distant flow-connected neighbour
OK … Point-kriging (Euclidean distance)

- Distribute weights according to distance only
- Topology not taken into account!!
- Too much weight according to distance in geographic space, and too little weight according to river network topology

1D Models … Point-kriging using stream distance

Weights (upstream model)

- All weight given to flow-connected neighbour and no weight for flow-unconnected neighbour
- Prediction of source area by river mouth, rather than by next source

- Good results if most information at connected sites
- Overall too much weight according to topology and too little weight according to distance in geogr. space

2D-Models … Top-kriging

- Distribute weights according to distance and river network topology, depending on data situation

Case study 1: 1D-modelling of environmental variables

- Garreta et al. (2009)
- 141 nitrate and 187 temperature stations
- Situated at the Meuse and Moselle basin in north-eastern France.

Case study 1: 1D-modelling of environmental variables

Results (Garreta et al. 2009)

• Summer temperature: the Tail-up model performed better
• Nitrate: the inverse was true
• A hybrid model which (= combination Tail-up & Tail-down) performed significantly better than each of the models separately.


Nitrate loads, Hybrid model: Prediction errors (left) and confidence interval (right)

• Segments without observation have significantly higher estimation errors (60%) than segments with observations (10% of obs. value)
• Abrupt change in between
  ➔ Reliable in the interpolation case
  ➔ Not reliable in the extrapolation case


Case study 2: 2D-modelling of low streamflows

Data: Austria, 491 gauges

Prediction

Top-kriging

Uncertainty (Kriging - standard error)

Top-kriging

- Performance increases with
  - gauging density
  - catchment size
  \[ \Rightarrow \text{Also less reliable in extrapolation case, but the effect is less pronounced (rmse}_{CV} = 2.4 \text{ and } 1.0 \text{ l/s/km}^2) \]

Case study 3: Annual stream temperature


Data:
- 214 gauges, mean annual streamflow temperature [°C]
... using altitude as external drift

\[
y = 11.487 e^{-0.0008 x}
\]

\[R^2 = 0.77\]

Data: 214 gauges

EDTK results: estimate

\[T^* = T(\text{Hmin})^* - \text{Resid}^*\]

Cross-validation

\[R^2 = 77\%\]

\[\text{rmse} = 1.01^\circ C\]

\[R^2 = 81\%\]

\[\text{rmse} = 0.80^\circ C\]
Regional examples
(1) Vorarlberg (western Austria)

External drift (Regression)  External drift Top-Kriging

Regional colder than expected from Hmin!

Top-Kriging corrects regional biases!

Conclusion

- We assessed geostatistical models for stream networks
- Ordinary-kriging (based on Euclidean distance) distribute weights according to distance only. Topology not taken into account!!
- 1D models give all weight to connected gauges at the same river, while close-by neighbors at unconnected rivers are not taken into account. Distribution of weights among tributaries is unsolved (Up-tail model).
- 2D models are more realistic; they distribute kriging weights according to spatial structure, distance and nestedness. They are consistent with hydrological concepts of runoff generation.
- Performance of 1D and 2D models was illustrated here in a meta-analysis of case studies. It would be interesting to perform a direct comparison on a common data set.

Thank you ...